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Topology of stirring in two-dimensional turbulence: Point vortex in a time-dependent ambient strain

Michał Branicki[1, *]

[1] *School of Mathematics, University of Bristol, University Walk, Bristol, BS8 1TW, UK*

We analyse the Lagrangian geometry of a time-dependent flow in the neighbourhood of a coherent vortex which is assumed to be a member of a vortex cluster collectively ruling the dynamics in the late stages of decaying two-dimensional turbulence. In order to gain insight into the highly inhomogeneous transport of a passive tracer in such vortex-dominated flows, we consider an idealised kinematic model of a local flow around a single coherent vortex in the cluster at distances much smaller than the distance to its nearest neighbour. We focus, in particular, on flow configurations which lead to a vigorous stirring and a subsequent escape of a passive tracer from the neighbourhood of the vortex. Here, the central vortex is approximated by a point vortex but the analytical arguments can be modified to cater for more realistic vorticity distributions. The principal axes of an irrotational ambient strain, which represents the combined, leading-order influence of its neighbours, are assumed to rotate with constant angular velocity and the strain-rate varies harmonically in time. The Lagrangian structure of the flow near the vortex is analysed by utilising the Hamiltonian formalism and employing appropriate perturbation methods. It is shown that sufficiently near the vortex there exist KAM-like tori which confine regions of purely chaotic tracer trajectories to the neighbourhood of the vortex. We emphasise, however, that there can exist certain 'open' flow geometries which lead to eventual 'leakage' of the tracer from 'sufficiently distant' regions of vigorous stirring to the outer flow. Such local flow configurations can be regarded as a prototype of the 'mixers' in decaying 2D turbulence.

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I. INTRODUCTION

The flow in the late stages of the decaying two-dimensional turbulence is dominated by a relatively small number of strong coherent vortices whose lifetimes are much longer than the characteristic time scale of the nonlinear turbulent interactions [3, 5, 13]. The analysis of velocity distributions show that these coherent structures destroy the self-similarity and the dynamical homogeneity of the transport dynamics at intermediate scales in the inertial range for active and passive tracers [2]. Even though it is known, based on the (local) tracer-gradient dynamics, that the passive tracer variance evolves from large to small scales the most rapidly within the robust 'elliptic' cores of the coherent vortices, many dynamical aspects of this process, and in particular differences between the passive and active case, remain unclear [1]. For example, in the Eulerian framework, the usually close-to-axisymmetric vortex cores are known to be robust to perturbations so that the vorticity filaments are stripped only from their edges during close interactions [14]. If then the passive tracer is vigorously mixed within the vortex core, what mechanisms are responsible for allowing it (or not) to escape to the outer flow?

Leaving the more intricate problem of active tracer dynamics aside, we approach the issue of passive scalar mixing (or rather stirring) in the absence of diffusion from the Lagrangian point of view, which lends itself to a variety of techniques from Dynamical Systems. This approach has been successfully used in the past in fluid dynamical considerations [16]; some of the most important results include the existence of KAM-like invariants, representing kinematical barriers to transport in time-periodic flows [10], robustness of Smale horseshoes [7] generating regions of chaotic fluid-particle trajectories, or lobe dynamics [19].

In the following sections, we study a simple kinematic model of a flow dominated by a nearly conservative mutual advection of coherent vortices, and assume that except for their brief and relatively infrequent interactions these structures move like interacting point vortices. Such idealised models were used successfully in the past in different contexts (see, for example, [13]). Here, we are particularly interested in the role each such a vortex plays in affecting the mixing (and motions) over extended regions of space. In particular, based on the robustness of hyperbolic sets, we establish the necessary conditions for existence of unbounded trajectories in a model flow, localised around a chosen vortex, leading to (locally) open-flow configurations. Other models of local transport, considering primarily stirring in bounded, chaotic regions in flows derived from time-periodic generalisations of the Kida's solutions, were also studied [8, 15]. Here, we also assume that the flow field is periodic in time which, while simplifying the analysis, reveals some interesting mechanisms for generation of non-trivial geometric templates for transport due to the central vortex. We note that, while introducing the time-dependence in this fashion may seem somewhat artificial, it proved to be very revealing in the past (see, for example, [4, 18, 20]).

*Electronic address: `m.branicki@bristol.ac.uk`

A more realistic model, allowing for aperiodic time dependence, will be discussed in a future publication. The more general treatment requires the use of lobe dynamics and geometric analysis of time-dependent invariant manifolds of the so-called Distinguished Hyperbolic Trajectories (the DHTs are also present in the time-periodic case but they are not necessary for the analysis) in the flow which 'spread the influence' of the DHTs globally throughout the flow (see, for example, [12] for a review).

II. EQUATIONS OF THE KINEMATIC MODEL

Consider an instantaneous flow, at a point (x, y) in Cartesian coordinates, due to a cluster of $N + 1$ point vortices located at (x_i, y_i), $i = 1, ..., N + 1$ and characterised by circulations Γ_i respectively. Provided that $(x, y) \neq (x_i, y_i)$, the corresponding vector field can be written as

$$\left.\begin{aligned} U(x,y) &= -\tfrac{1}{2\pi}\textstyle\sum_{i=1}^{N+1} \dfrac{\Gamma_i\cdot(y-y_i)}{(y-y_i)^2+(x-x_i)^2}\,, \\ V(x,y) &= \tfrac{1}{2\pi}\textstyle\sum_{i=1}^{N+1} \dfrac{\Gamma_i\cdot(x-x_i)}{(y-y_i)^2+(x-x_i)^2}\,, \end{aligned}\right\} \qquad (1)$$

where a positive circulation is assumed for a counter-clockwise rotating vortex. The flow in a neighbourhood of, say, the $(N + 1)$-st vortex due to the remaining N vortices can be easily derived by shifting the origin to the vortex (i.e. $x = x_{N+1} + \tilde{x}$, $y = y_{N+1} + \tilde{y}$) and summing (1) up to to the first $i = N$ terms.

A useful linearisation around the chosen vortex can be obtained by first transforming (1) to the polar coordinates (i.e. $x = \varrho\cos\varphi$, $y = \varrho\sin\varphi$) and then using the multipole expansion, which leads to the following representation of the velocity field

$$\begin{aligned} U(\varrho,\varphi) &= -\frac{1}{2\pi}\sum_{i=1}^{N}\frac{\Gamma_i}{\delta_i}\left(\frac{\varrho}{\delta_i}\sin\varphi-\sin\phi_i\right)\times \qquad (2)\\ &\times\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left(\frac{\varrho}{\delta_i}\right)^{k+l} P_l\left(\cos(\varphi-\phi_i)\right)P_k\left(\cos(\varphi-\phi_i)\right), \end{aligned}$$

$$\begin{aligned} V(\varrho,\varphi) &= \frac{1}{2\pi}\sum_{i=1}^{N}\frac{\Gamma_i}{\delta_i}\left(\frac{\varrho}{\delta_i}\cos\varphi-\cos\phi_i\right)\times \qquad (3)\\ &\times\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left(\frac{\varrho}{\delta_i}\right)^{k+l} P_l\left(\cos(\varphi-\phi_i)\right)P_k\left(\cos(\varphi-\phi_i)\right); \end{aligned}$$

here $(x_i, y_i) = (\delta_i\cos\phi_i, \delta_i\sin\phi_i)$ and P_l denotes the Legendre polynomial of the l-th order. Denoting the distance to the nearest neighbour by $\delta_{min} = \min\{\delta_i\} \neq 0$, we can now linearise the flow in the region $\varrho/\delta_{min} \ll 1$ as follows:

$$\begin{aligned} U \sim &\sum_{i=1}^{N}\frac{\Gamma_i\sin\phi_i}{2\pi\delta_i}+\varrho\cos\varphi\sum_{i=1}^{N}\frac{\Gamma_i\sin 2\phi_i}{2\pi\delta_i^2}\\ &-\varrho\sin\varphi\sum_{i=1}^{N}\frac{\Gamma_i\cos 2\phi_i}{2\pi\delta_i^2}+\mathcal{O}\left(\varrho^2/\delta_{min}^2\right), \qquad (4)\end{aligned}$$

$$\begin{aligned} V \sim &-\sum_{i=1}^{N}\frac{\Gamma_i\cos\phi_i}{2\pi\delta_i}-\varrho\cos\varphi\sum_{i=1}^{N}\frac{\Gamma_i\cos 2\phi_i}{2\pi\delta_i^2}\\ &-\varrho\sin\varphi\sum_{i=1}^{N}\frac{\Gamma_i\sin 2\phi_i}{2\pi\delta_i^2}+\mathcal{O}\left(\varrho^2/\delta_{min}^2\right). \qquad (5)\end{aligned}$$

Clearly, the ϱ^0 terms in (4, 5) correspond to a uniform field driving the vortex core. The ϱ^1 terms represent the first order approximation to the deformation field which can be transformed back to the Cartesian coordinates in the form

$$\begin{bmatrix} U^1 \\ V^1 \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ -\beta & -\alpha \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}, \quad \begin{aligned}\alpha &= \sum_{i=1}^{N}\Gamma_i\sin 2\phi_i/2\pi\delta_i^2,\\ \beta &= \sum_{i=1}^{N}\Gamma_i\cos 2\phi_i/2\pi\delta_i^2\,.\end{aligned} \qquad (6)$$

The flow given by (6) can be identified as an irrotational strain with (orthogonal) principal axes rotated with respect to $\hat{\mathbf{e}}_x$ by an angle $\theta = \mathrm{atan}(\beta/\alpha)$ and amplitude $\mathcal{A} = (\alpha^2+\beta^2)^{1/2}$. Evolution of the vortex cluster implies that both the amplitude and orientation of the ambient strain vary in time (since $\delta_i = \delta_i(t)$, $\phi_i = \phi_i(t)$). We model it simply by assuming that the strain axes rotate with constant angular velocity λ, and that the strain amplitude depends harmonically on time,

$$\hat{\mathcal{S}}\, s_{\omega,\Delta}(t) = \hat{\mathcal{S}}\frac{\Delta+\cos(\omega t)}{\Delta+1}\,. \qquad (7)$$

We stress, however, that as long as the vortex model remains axisymmetric and the strain is linear and time-periodic, similar analysis to that discussed below can be carried out.

If we now choose as a unit of time $T = 1/\hat{\mathcal{S}}$ and unit of length $L = (\Gamma/2\pi\hat{\mathcal{S}})^{1/2}$, the local (non-dimensionalised) flow around a point vortex in the frame of reference rotating with the axes of the strain can be written as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{1}{(x^2+y^2)}\begin{bmatrix} -y \\ x \end{bmatrix} + \begin{bmatrix} -s_{\Omega,\Delta} & \Lambda \\ -\Lambda & s_{\Omega,\Delta} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}, \qquad (8)$$

where now $\Omega^{-1} = \hat{\mathcal{S}}/\omega$ is the dimensionless frequency of oscillations of the strain amplitude, and $\Lambda = \lambda/\hat{\mathcal{S}}$ is the angular velocity of the strain axes. Finally, the system (8) can be re-written in polar coordinates as

$$\left.\begin{aligned} \dot{\varrho} &= -s_{\Omega,\Delta}\,\varrho\cos 2\varphi, \\ \dot{\varphi} &= s_{\Omega,\Delta}\sin 2\varphi-\Lambda+1/\varrho^2, \end{aligned}\right\} \qquad (9)$$

which will prove useful in analysis presented in §IV.

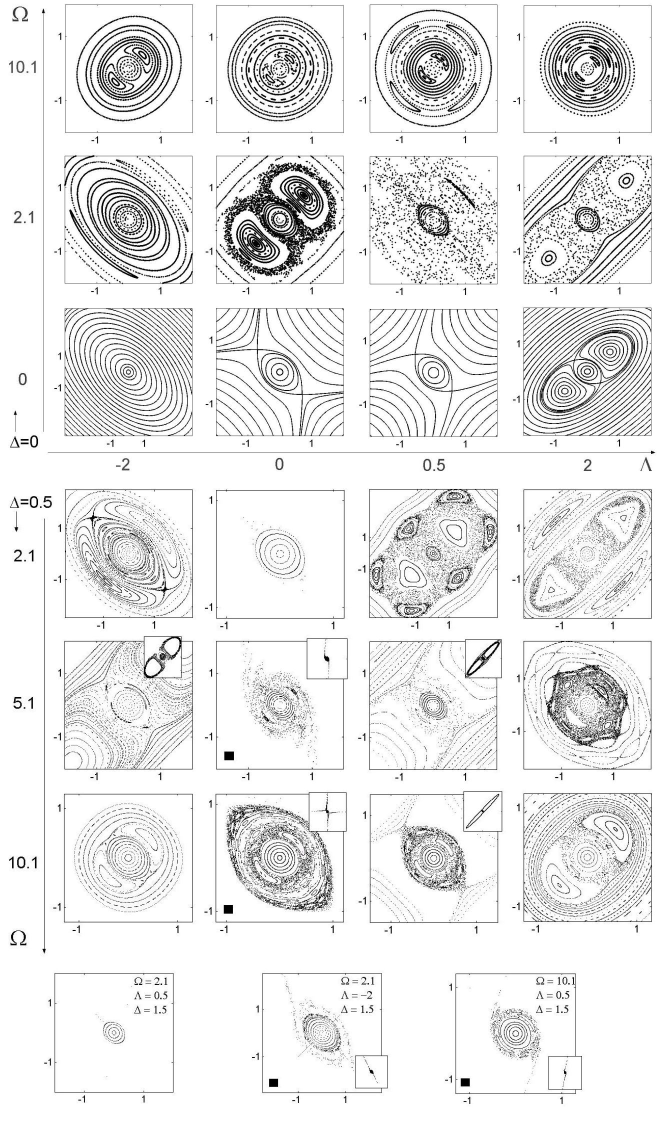


FIG. 1: Examples of Poincaré sections for advection by the flow (9) in the frame rotating with the strain axes for different values of the system parameters $(\Omega, \Lambda, \Delta)$. Note in particular the examples of 'open' flow configurations, which are characterised by the existence of a set of initial conditions (of non-zero measure) giving rise to trajectories which evolve in the area of rapid mixing for a finite time and then escape to the outer flow. The condition for existence of such configurations is given by (18).

III. LAGRANGIAN VS. EULERIAN VIEWPOINTS

It is instructive to analyse first the structure of the system (9) in the steady case (i.e. $s_{\Omega,\Delta} = \text{const.}$) depending on the system parameters. One can easily deduce that fixed points of (9) lie symmetrically with respect to the origin on one of diagonal lines inclined at $\pm 45^\circ$ to the strain axes. Because of the inherent symmetry in the model, there is always an elliptic fixed point at the centre, which is also the only stagnation point in the flow for $\Lambda < -1$. For $-1 < \Lambda < 1$ there are additionally two hyperbolic fixed points located at

$$\varrho^{\text{hyp}}_{1,2} = (\Lambda + 1)^{-1/2}, \quad \varphi_1 = -\pi/4, \quad \varphi_2 = 3\pi/4\,, \quad (10)$$

and the system trajectories trace out the familiar cat's eye pattern (see figure 1 for $\Omega = 0$) where the two heteroclinic connections confine the recirculating flow to the neighbourhood of the origin. For $\Lambda > 1$ two more elliptic fixed points are present in the flow, and are located at

$$\varrho^{\text{ell}}_{1,2} = (\Lambda - 1)^{-1/2}, \quad \varphi_1 = \pi/4, \quad \varphi_2 = -3\pi/4\,. \quad (11)$$

In this case, there are four distinct regions in the flow which all contain bounded (in fact periodic and non-isolated) trajectories. These regions are separated by invariant manifolds of the hyperbolic fixed points which implies that tracers located initially in different regions do not mix.

The above scenario can be dramatically altered when the strain amplitude, $s_{\Omega,\Delta}$, is time-dependent (i.e. when $\Omega \neq 0$) so that the location of ISP's (instantaneous stagnation points the vector field (9) at $t = t^*$) changes in time. It is important to recognise that the ISP's are frame-dependent, Eulerian objects and that their paths in the extended phase space (spanned in this case by two space coordinates and time) are no longer trajectories of the system (9). One can easily deduce that the ISP's must lie on the same diagonal lines as in the steady case but now the distance to the origin is given by

$$\begin{cases} \varrho^{\text{hyp}}_{1,2}(t) = \big(\Lambda + s_{\Omega,\Delta}(t)\big)^{-1/2} & \text{if} \quad \Lambda > -s_{\Omega,\Delta}, \\ \varrho^{\text{ell}}_{1,2}(t) = \big(\Lambda - s_{\Omega,\Delta}(t)\big)^{-1/2} & \text{if} \quad \Lambda > \ \ s_{\Omega,\Delta}. \end{cases} \quad (12)$$

In the unsteady case, the 'Eulerian' snapshots, given by the instantaneous streamline patterns, bear little correspondence to the actual structure of trajectories in the system's phase space which, for time-periodic flows, can be studied by means of the 2D, area-preserving Poincaré map.

A few examples of such a mapping, shown in figure 1 for a different system parameter values, give a flavour of the complexity introduced into the trajectory structure by the simple time-periodic strain variations. In a generic case, the time-dependence introduces a wealth of chaotic regions which are often separated (in 2D) by robust KAM-like invariants. If all contours of these invariants are closed, the spatial extent of the mixing by the vortex is limited. If, however, such invariants do not exist at sufficiently large distances from the origin or simply have an 'open' topology (see figure 2b), there can exist a set of initial conditions of non-zero measure which yields trajectories that reside in a region of vigorous stirring near the vortex for a finite time and then escape to the outer flow (see bottom of figure 1).

It can be shown, using the Kruskal's averaging method [9], that when all trajectories of the time-dependent ambient strain are periodic (for example $\Delta = \Lambda = 0$), there exist adiabatic invariants in the nonlinear system (9) which prevent the tracer from escaping the neighbourhood of the vortex [6]. The problem of finding such an adiabatic invariant structure becomes formidable in less degenerate cases (for example when either $\tilde{s}$ or $\tilde{\lambda}$ is aperiodic in time) and we defer such analysis to a future publication. Instead, we note that sufficiently far away from the origin, the flow due to the vortex can be regarded as a perturbation to the base-flow of the time-dependent ambient strain (the last term in (8)), which is fully characterised by the fundamental solution matrix

$$\Phi(t,t_0) = \begin{bmatrix} \cosh\tilde{\sigma} - (\tilde{s}/\tilde{\sigma})\sinh\tilde{\sigma} & -(\tilde{\lambda}/\tilde{\sigma})\sinh\tilde{\sigma} \\ (\tilde{\lambda}/\tilde{\sigma})\sinh\tilde{\sigma} & \cosh\tilde{\sigma} + (\tilde{s}/\tilde{\sigma})\sinh\tilde{\sigma} \end{bmatrix} \tag{13}$$

where

$$\tilde{s} = \int_{t_0}^{t} s(t')\mathrm{d}t', \quad \tilde{\lambda} = \int_{t_0}^{t} \Lambda(t')\mathrm{d}t', \quad \tilde{\sigma} = \sqrt{\tilde{s}^2 - \tilde{\Lambda}^2}. \tag{14}$$

Using (13), we can relatively easily check for the existence of unbounded hyperbolic trajectories in the unperturbed system (8). (Hyperbolic trajectories and their invariant manifolds bear similarities to non-degenerate saddle fixed points (and their manifolds) in the sense of how nearby trajectories behave in the frame of reference moving with the hyperbolic trajectory [21].) Due to the robustness of hyperbolic structures (see [17] and references therein), we can deduce that for any hyperbolic trajectory of (13) such that

$$\|\boldsymbol{\Phi}(t,t_0)\boldsymbol{x}_0\| \gg 1, \forall\, t \in [t_0,\ \infty), \tag{15}$$

there exists a 'perturbed' analogue in the flow (9). Based on the form of (13), we can easily see that in order for a trajectory of the unperturbed linear flow to be unbounded and hyperbolic, there must exist a time t^* such that

$$\Re[\sigma] \neq 0,\ \Re[\mathrm{d}\sigma/\mathrm{d}t] > 0,\ \forall \quad t > t^*, \ \wedge\ \ \mathcal{L}_{1,2} \neq 0, \tag{16}$$

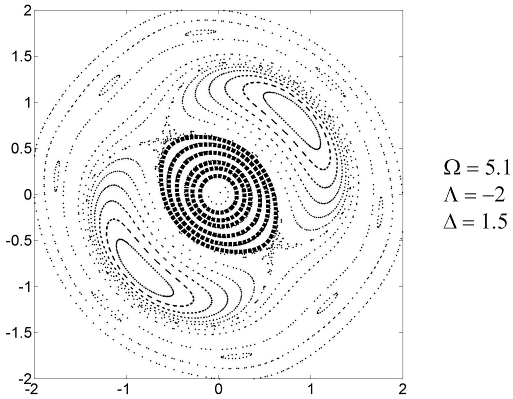
where the Lyapunov exponents, $\mathcal{L}_{1,2}$, are defined in the standard way as the logarithms of the eigenvalues of

$$\mathcal{M} = \lim_{(t-t_0)\to\infty} (\boldsymbol{\Phi}^T(t,t_0)\boldsymbol{\Phi}(t,t_0))^{1/2(t-t_0)}\,. \tag{17}$$

In the case of our model, the conditions (16) reduce to

$$\begin{cases} \Delta^2 - \Lambda^2(\Delta+1)^2 > 0 & \text{if} \quad \Omega \neq 0, \\ \Lambda^2 < 1 & \text{if} \quad \Omega = 0. \end{cases} \tag{18}$$

It should be noted, however, that the criterion (18), remains rather formal in the sense that we do not specify explicitly the set of initial conditions that satisfies (15); this becomes particularly tricky for $\Omega \to 0$ when, due to the unbounded oscillatory character of the unperturbed strain trajectories, even distant trajectories may return close to the origin.



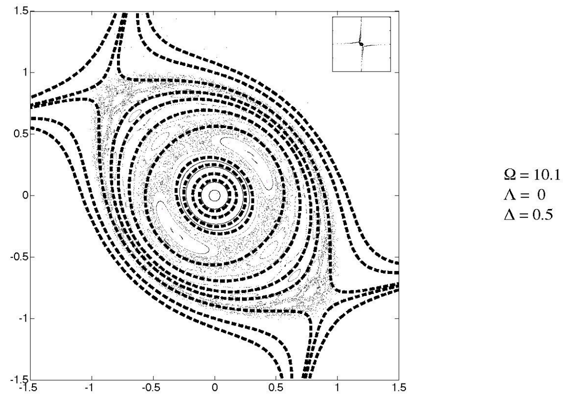


FIG. 2: Comparison between the numerically computed, nearly-integrable structure of Lagrangian fluid trajectories of the flow (9) at times $t_n = 2\pi n/\Omega$ (dots), and the contours of an approximate invariant (26) derived using time-dependent Hamiltonian perturbation method (thick dashed lines). In the second example, even the 'open' invariant structure is surprisingly well approximated.

IV. 'NEAR-FIELD' STRUCTURE

As seen from Poincaré sections shown in figure 1, the structure of the system trajectories near the origin resembles that of a nearly-integrable Hamiltonian system, and the KAM invariants prevent the tracer trapped too close to the vortex core from escaping. Consequently, even in the open flow configurations discussed in the previous section only the trajectories starting sufficiently far away from the origin can be stripped away from the vortex. In order to gain more insight into the geometry of those impenetrable mixing barriers, we can determine an approximate (integrable) phase-space structure near the vortex by employing the Hamiltonian perturbation theory, which we outline here only briefly (see [11] for more details). We note that the system (9) can be re-written in the Hamiltonian form, i.e.

$$\dot{J} = -\partial H/\partial\theta, \quad \dot{\theta} = \partial H/\partial J, \tag{19}$$

with the Hamiltonian written in the 'perturbed' form

$$\begin{aligned} H(J,\theta,t) &= H^0(J) + \epsilon H^1(J,\theta,t) \\ &= -\frac{1}{2}\ln J + J\Lambda + \epsilon J s_{\Omega,\Lambda}\sin 2\theta, \end{aligned} \tag{20}$$

where $J = \varrho^2/2$, and $\theta = -\varphi$. The first term in (20) corresponds to the Hamiltonian of the point-vortex flow and the remaining terms represent the Hamiltonian of the time-dependent strain. We note that sufficiently near the origin, i.e. $J \ll 1$, the 'non-rotating' part of the ambient strain can be treated as a perturbation, which is non-uniform in space. We emphasize this fact by introducing an 'ordering' parameter - ϵ - which will be later set to unity. The perturbation H^1 is periodic in t, with period $T = 2\pi/\Omega$, and in θ, with period 2π, and can be expanded in the Fourier series as

$$H^1 = \sum_{lm} \hat{H}^1_{lm}(J)e^{\mathrm{i}(l\theta + m\Omega t)}. \tag{21}$$

It can be shown (see [11]) that for the perturbed, non-autonomous, one-degree-of-freedom Hamiltonian system (20), there exists a 'nearby' integrable Hamiltonian system is given by

$$\bar{H}(\bar{J}(J,\theta)) = H^0(J) + \epsilon\langle H^1\rangle_{\theta,t}\,, \tag{22}$$

where $\langle\cdot\rangle_{\theta,t}$ denotes a 'fast-variable' average over the 'fast' manifold (torus in this case). The new invariant, $\bar{J}(J,\theta,t)$, is obtained by seeking a near-identity generating function

$$S(\bar{J},\theta) = \bar{J}\theta + \epsilon S_1(\bar{J},\theta,t) + O(\epsilon^2), \tag{23}$$

which, when combined with (22), enables determination of S_1 (up to $\mathcal{O}(\epsilon)$) in the form

$$S_1 = i\sum_{\substack{lm \\ l=m\neq 0}} \frac{H^1_{lm}(J)}{l\omega(J) + m\Omega} e^{i(l\theta + m\Omega t)}\,, \tag{24}$$

and subsequently yields the new invariant in the form

$$\bar{J} = J - \epsilon\frac{\partial S_1(J,\theta,t)}{\partial\theta}\,. \tag{25}$$

In the particular case when the perturbing Hamiltonian, H^1, is given by the last term of (20), the new invariant (25) can be written as

$$\bar{J} = J + \frac{\epsilon J}{\Delta + 1}\left[\frac{\Delta\sin(2\theta)}{\omega(J)} + \frac{\sin(2\theta + \Omega t)}{2\omega(J) + \Omega} + \frac{\sin(2\theta - \Omega t)}{2\omega(J) - \Omega}\right] \tag{26}$$

where $\omega(J) = \partial H^0/\partial J = -1/(2J) + \Lambda$, is the winding frequency on the unperturbed torus. The approximation (26) is obviously valid away from the resonances given by $\omega(J) = \pm\Omega/2$ and $\omega(J) = 0$.

We compare (26) with direct integration of the system (9) in figure 2 again by computing the Poincaré sections. The contours of the new invariant clearly are a very good approximation to the integrable structure near the origin. It is often the case that approximations obtained via perturbation techniques are a surprisingly good approximation well beyond the range of their formal applicability. One can observe a similar effect here: the integrable structure is often reasonably well approximated even at distances $\varrho \sim \mathcal{O}(1)$ although the chaotic separatrix layers are obviously missed by the approximation. Note in particular the open contours of the new invariant in figure 2b which allow trajectories in the outermost chaotic band to escape.

V. CONCLUSIONS

We studied the Lagrangian structure of a simple, time-periodic model flow in an attempt to elucidate the mechanisms responsible for stirring of a passive scalar around strong, coherent vortices in advection-dominated flows. We showed, using time-dependent Hamiltonian perturbation method, that the chaotic regions located sufficiently near the centre of the vortex are bounded by KAM-like invariants which serve as barriers to mixing, trapping the tracer near the vortex. However, there exist flow configurations where, even in the absence of close interactions with neighbouring vortices, some tracer is stripped off the the vortex after a transient period of being vigorously mixed in its neighbourhood. This framework can be easily extended to more realistic models of the vortex core, as long as it remains axisymmetric and the ambient strain is time-periodic. Further analysis is clearly needed in order to investigate the consequences of aperiodic time dependence.

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